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a speed of revolution of over 1000 turns per second has been attained. This makes it possible to obtain a rotation of 45° while light is passing twice over the distance between stations $12^1/_2$ miles apart; or of 90° if the stations are 25 miles apart.

It might be thought that with so small a mirror face the intensity of the return image would be insufficient; but by placing the revolving mirror in the focus of a very long focus lens or mirror, and placing the slit source at a distance $\nu = f^2/D$, in which f is the focal length and D the distance between stations, the intensity is a maximum and yet a small mirror is sufficient to return all the light which enters the lens. This last (or the concave mirror which replaces it) should be as large as possible as also, of course, the distant mirror.

Incidentally the eight reflections from the revolving mirror, will give four times as much light as a two-faced plane mirror.

Preparations are now in progress for a preliminary test of the method at Mt. Wilson, where it is hoped a distance of 25 miles may be utilized.

ON THE K SERIES OF X-RAYS

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Communicated July 1, 1920

In the research reported in this paper the authors have measured the wave-lengths of the emission lines and that of the critical absorption in the K series of tungsten. In order to obtain as precise values of the wave-lengths as possible they employed spectra of the first, second, third, fourth and, in one instance, the fifth order.

The object of the research has been to provide data for testing the following points: (*a*) The existence of a third line in the α-group; (*b*) the separation of the critical absorption from the line of shortest wave-length in the emission spectrum, namely the γ line; (*c*) the experimental and theoretical relations between the various lines in the K, L, M, etc., series; (*d*) the relative intensities of the emission lines; and (*e*) the equations for the wave-lengths that may be deduced from theories of the structure of atoms and the mechanism of radiation.

The X-ray spectrometer, the generating plant used to operate the X-ray tube, the instruments for controlling the current and voltage and the general method of making the measurements have been described in papers from this laboratory published during the last few years in the

Physical Review. The chief difference between this and previous researches lies in fact that we have used spectra of higher orders, thus obtaining greater dispersion and precision.

Measurements of the electric current in the ionization chamber of the spectrometer were made at intervals of 30″ of arc on the scale which gives the grazing angle of incidence of the X-rays that strike the crystal. The curves in figures 1, 2 and 3 represent the ionization current as a function of the grazing angle of incidence, θ, in typical experiments.

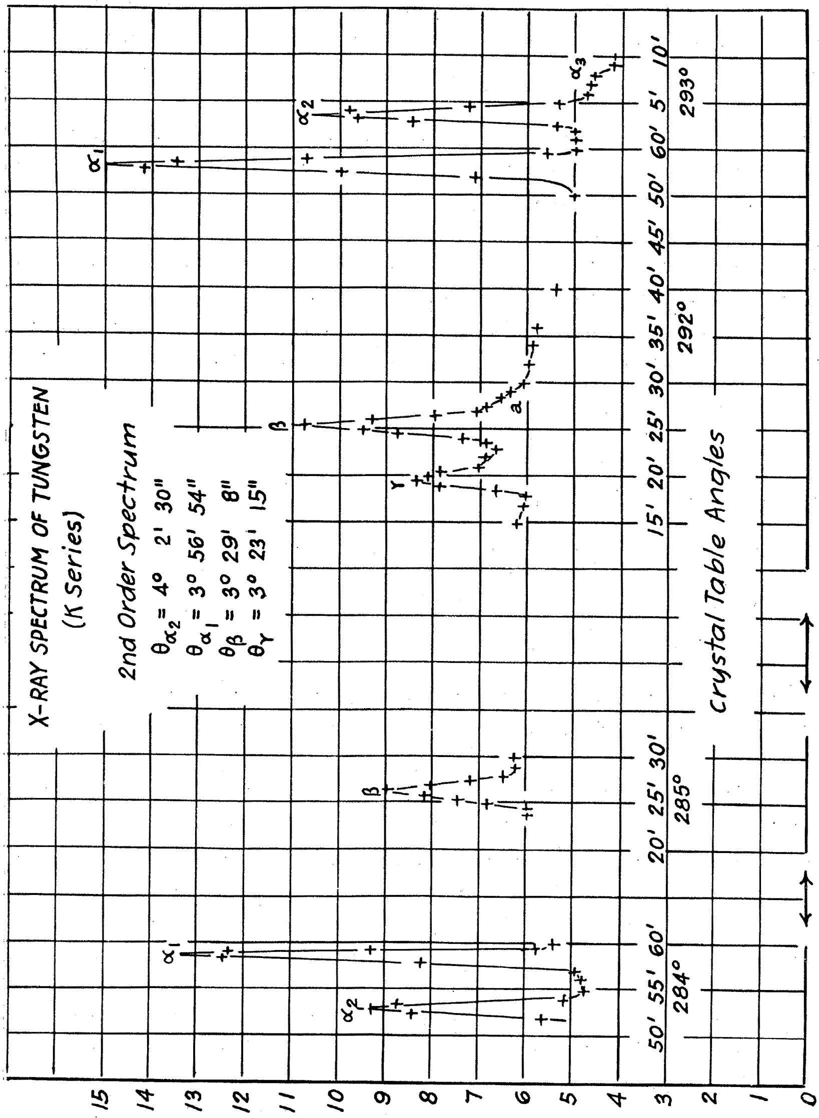

FIG. 1.

The tips of the peaks in these curves correspond to the lines in the characteristic X-ray spectra. It is possible to determine with certainty between which two settings of the crystal a peak lies, and also about how far from each it is. This means that, when the apparatus is functioning well, the grazing angle of incidence can be estimated to within about 7″ of arc. For the tungsten K series the grazing angles of incidence in the first order spectrum amount to about 2° = 7200″, and, therefore, the error in a satisfactory determination of a wave-length ought not to exceed $^1/_{10}$, $^1/_{20}$, $^1/_{30}$, $^1/_{40}$, etc., per cent in the spectra of the first, second, third, fourth, etc., orders, respectively.

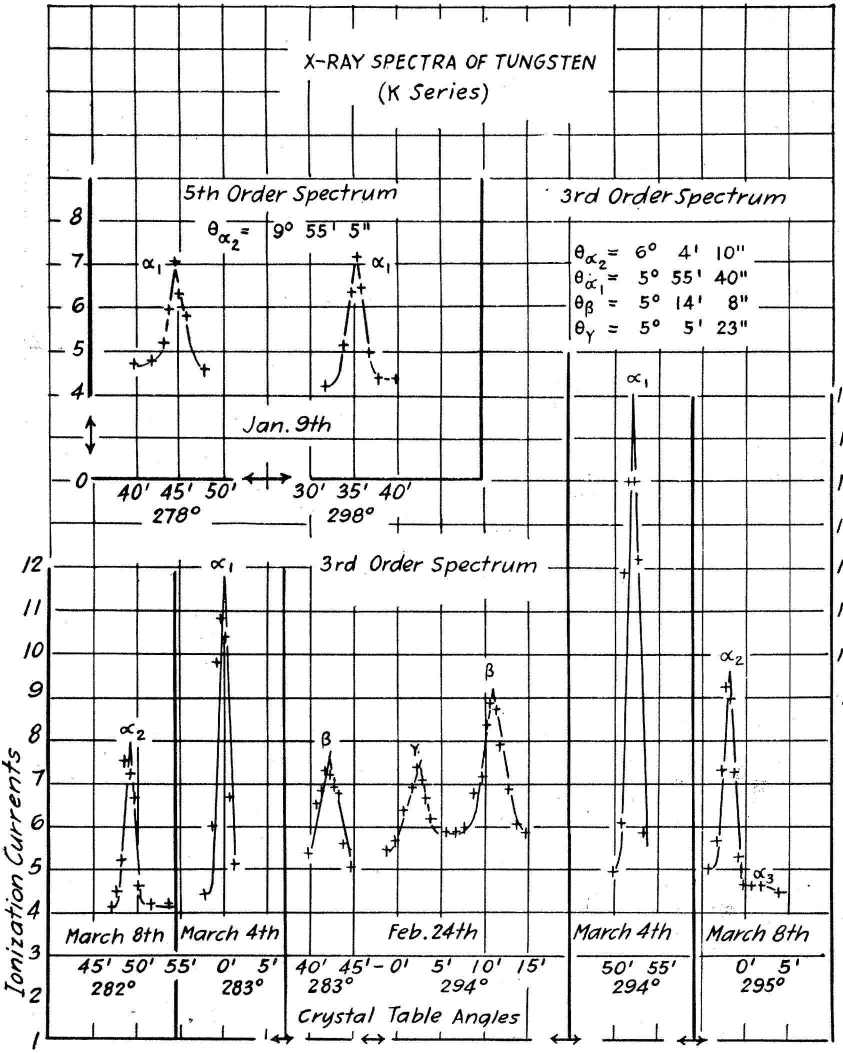

FIG. 2

In estimating the wave-lengths, etc., only those measurements may be used that are made on the same date, for the zero of the instrument may have shifted between the dates recorded in the figures.

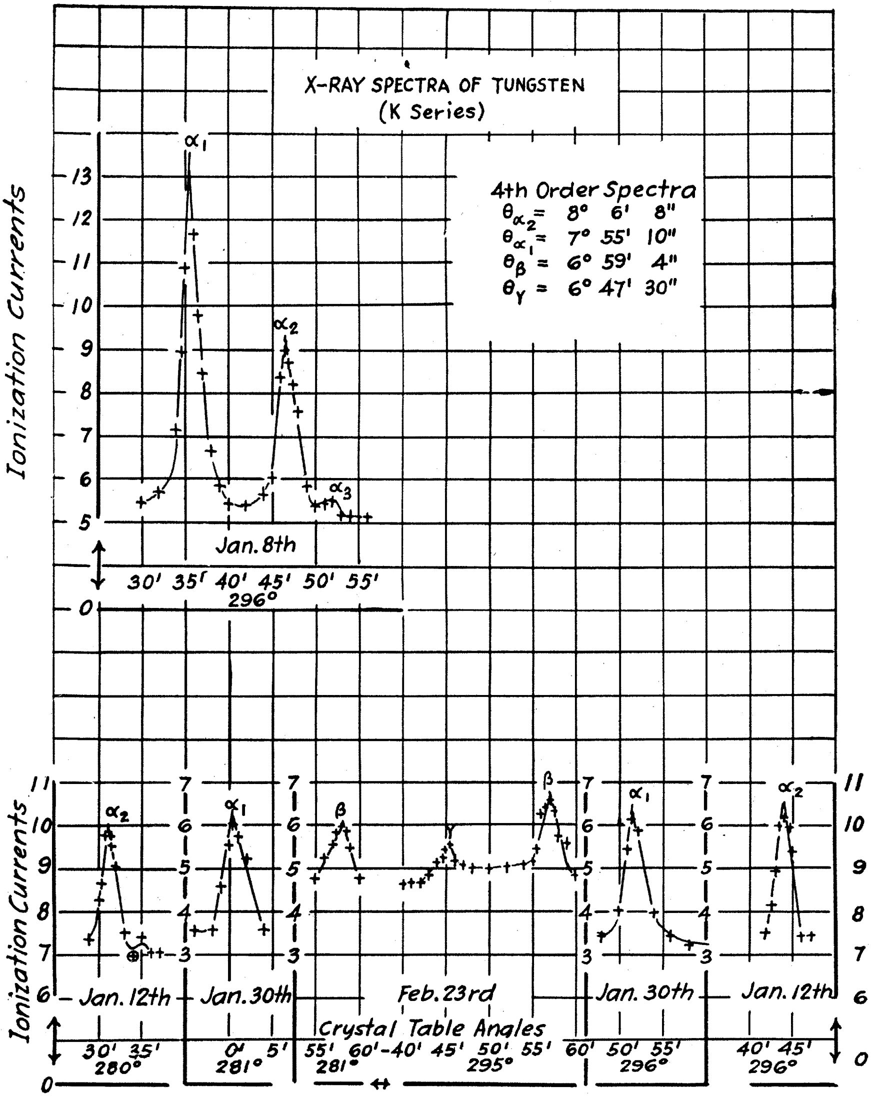

FIG. 3

The following table contains the wave-lengths, λ, calculated from the average grazing angles of incidence, θ, obtained from all the measurements we have made in the spectra of the various orders, respectively.

TABLE 1

K SERIES OF TUNGSTEN

$n\lambda = 6.056 \times \sin\theta \times 10^{-8}$ cm.

ORDER, n, OF SPECTRUM	$\lambda \times 10^8$ cm.					
	α_3	α_2	α_1	β	γ	ABSORPTION
1				0.18441	0.17898	0.17833
2		0.21342	0.20850	0.18415	0.17880	0.17810
3		0.21339	0.20848	0.18420	0.17909	0.17795
4	0.215	0.21341	0.20862	0.18418	0.17905	
5			0.20862			
Weighted Means	0.215±1	0.21341±3	0.20860±4	0.18420±3	0.17901±6	0.17806±7
Last Year		0.2134	0.2087	0.1842		0.1785
Siegbahn		0.21345	0.20878	0.18430	0.17934	
$\frac{1}{\lambda}$	4.65	4.6858	4.7938	5.4290	5.5863	5.6160
$\frac{\nu}{\nu_\infty}$	4239.	4270.3	4368.7	4947.4	5090.9	5118.0

The seventh line in the table contains the weighted mean values of the wave-lengths, together with a rough estimate of the precision of the measurements. The γ wave-length is more difficult to measure than the others owing to the fact that the critical absorption wave-length lies so near it. The target of the X-ray tube absorbs some of its own rays.

In order to estimate the absolute accuracy of the wave-lengths we must, of course, as in all work in X-ray spectra, take account of the errors in the grating constant of the reflecting crystal. These appear to add up to about 0.06%. (See a "Report on Data Relating to X-Ray Spectra," published by the National Research Council.)

For purposes of comparison we have added, in the eighth and ninth lines, respectively, the values obtained last year by the ionization method (*Physical Review*, July, 1919, p. 67) and those given by Siegbahn (*Phil. Mag.*, Nov., 1919, p. 639), which he measured by photographic methods. A small correction ($^1/_{3000}$) has been made in these wave-lengths corresponding to the value of the grating constant of calcite we use.

In order to test general relations between the wave-lengths and also theoretical equations it is convenient to have at hand the wave-numbers, $1/\lambda$, and the ratios, ν/ν_∞, of the frequencies, ν, to the fundamental Rydberg frequency, ν_∞. In calculating these ratios we have used the Rydberg fundamental wave-number, $\nu_\infty = 109737$, calculated for heavy atoms from the data obtained by Paschen in the spectra of hydrogen and helium. Line 10 in table 1 contains the values of $1/\lambda$, each divided by 10^8, and line 11 contains the values of ν/ν_∞, calculated from our values of λ.

The critical absorption wave-lengths appear in the last column of table

1. Dr. R. A. Patterson coöperated with us in making the measurements of these wave-lengths. We used an X-ray tube with a molybdenum target in these experiments.

It appears from the data that the wave-length of the γ line and that of the critical absorption differ from each other by about $^1/_2$ of 1%. In the K series spectrum of rhodium (see *Physical Review*, Oct., 1919, p. 369) the two wave-lengths differ from each other by about $^1/_4$%. If we adopt the theory of electrons revolving in atomic orbits we can explain this difference in wave-length as follows: According to the theory the critical absorption frequency equals the amount of energy required to carry an electron from the K orbit out to the periphery of the atom divided by Planck's constant, h. The γ line is due to electrons falling into the vacancies in the K orbit from orbits outside of those that we may class together and call the M orbits. The amount of energy set free and radiated during one of these transfers of an electron is less than the amount of energy required to lift the electron all the way from the K orbit to the periphery of the atom. Since the frequency of the X-ray emitted during the transfer equals this energy divided by h, it follows that the frequency must be less than that of the critical absorption, and, therefore, its wave-length greater than that of the critical absorption. The γ line is probably complex, and the wave-length we measure must be a kind of centre of gravity of the wave-lengths of its components.

In the second order spectrum (fig. 1) a drop occurs in the curve at a certain point marked "*a*." This does not belong to the K series of tungsten. It represents the critical ionization of iodine in the first order spectrum, the wave-length of which is 0.3737×10^{-8} cm. We used methyliodide in the ionization chamber of the spectrometer.

In a research described in the *Physical Review* for July, 1919, p. 67, Mr. Shimizu and one of us obtained experimental evidence showing that the difference between the K critical absorption frequency and any one of the L critical absorption frequencies equals the frequency of one of the emission lines in the K series. Theoretically this relation ought not to be exact, if orbits in some of the atoms are elliptical and in other atoms of the same chemical element, circular. The atoms with elliptical orbits contain amounts of energy that are different from the amounts of energy in atoms with circular orbits, and hence the change in energy when an electron is removed from the K orbit will not be the same in the two cases.[1] This means that the K critical absorption frequency should be complex, and that what we measure is a kind of average frequency. Strictly, according to the theory, the frequency difference law should apply to those atoms only that are exactly alike.

To test this point we have taken the absorption measurements made on the L series of tungsten by Dr. R. A. Patterson and one of us. The law applies, of course, to the wave-numbers as well as to the frequencies.

The wave-numbers of the three critical absorptions in the L series of tungsten are

$$\mathrm{La}_1 = 0.8240 \pm 1 \quad \mathrm{La}_2 = 0.9323 \pm 4 \quad \mathrm{La}_3 = 0.9766 \pm 30,$$

each multiplied by 10^8. Subtracting these from the critical absorption wave-number in the K series and comparing the differences with the wave-numbers of the three α emission lines we get

WAVE-NUMBERS, $1/\lambda \times 10^{-8}$

$\mathrm{Ka} - \mathrm{La}_1 = 4.792 \pm 0.003$ $\quad$ $\mathrm{K}\alpha_1 = 4.7938 \pm 0.001$
$\mathrm{Ka} - \mathrm{La}_2 = 4.685 \pm 0.003$ $\quad$ $\mathrm{K}\alpha_2 = 4.6858 \pm 0.0007$
$\mathrm{Ka} - \mathrm{La}_3 = 4.639 \pm 0.006$ $\quad$ $\mathrm{K}\alpha_3 = 4.65 \pm 0.02$

It appears that in each case the difference between the K critical absorption wave-number and one of the L critical absorption wave-numbers equals the wave-number of one of the α emission lines to within the limits of error of the measurements. This agrees with the results obtained last year. If the above mentioned effect due to elliptic orbits exists at all, it appears to be too small to detect, with our present methods of measurements, in atoms of as high an atomic number as that of tungsten.

The existence of the third faint line, α_3, and the agreement of its wave-number with the difference between the Ka and La_3 wave-numbers is a point of some theoretical interest. According to Sommerfeld's theory of elliptic orbits the $\mathrm{K}\alpha_1$ line is due to the transfer of an electron from the L orbit to the K orbit, if the L orbit is circular, and the $\mathrm{K}\alpha_2$ line is due to a similar transfer, if the L orbit is elliptic. The difference between the wave-numbers of $\mathrm{K}\alpha_1$ and $\mathrm{K}\alpha_2$ calculated from the formulas he gives (*Atombau und Spektrallinien*, chapter 5) amounts to 0.1072×10^8, and this agrees very well with our experimental value for the same difference, namely, 0.1080×10^8. There appears to be no explanation on this theory for the third critical absorption, La_3, in the L series, nor for the third line $\mathrm{K}\alpha_3$ in the K emission series. It would seem to be necessary to assume that there are at least two L orbits.

The peak corresponding to $\mathrm{K}\alpha_3$ seems to be quite well marked on the curves representing spectra of higher order than the first, especially on that representing the fourth order. The line is very faint, having only about $^1/_{25}$ of the intensity of the $\mathrm{K}\alpha_1$ line.

The $\mathrm{K}\beta$ line is supposed to be due to electrons falling from the M orbit (or orbits) into the K orbit. We can not test the above frequency difference law in this case directly, for the critical absorption frequencies have not been measured in the M series of tungsten. Dr. Stenström, however, has observed three critical absorption wave-lengths in the M series of thorium and uranium. He has measured also the wave-lengths of emission lines in the M series of uranium, thorium and tungsten. Since the relative positions of the critical absorption and certain of the emission lines are about the same in the high frequency spectra of the different chemical elements, we can estimate approximately where the critical absorption wave-lengths lie in the M series of tungsten. Making this estimate, and calculating the three critical absorption wave-numbers we obtain the following values, each multiplied by 10^8,

$$Ma_1 = 0.152 \quad Ma_2 = 0.157 \quad Ma_3 = 0.182.$$

Subtracting these from the K absorption wave-number, and comparing the differences with the Kβ emission wave-number we get

$$Ka - Ma_1 = 5.464$$
$$Ka - Ma_2 = 5.459 \qquad K\beta = 5.429$$
$$Ka - Ma_3 = 5.434$$

One of the differences between absorption wave-numbers, $Ka - Ma_3$ lies very close to that of the Kβ line. This suggests that the Kβ line is due chiefly to electrons falling from the third M orbit to the K orbit. If electrons also fell from the second M orbit to the K orbit they would produce a line on the short wave-length side of the principle Kβ line. Faint lines on the short wave-length sides of the Kβ lines have been observed in the spectra of a few of the chemical elements of low atomic number by Mr. Hjalmar working in Professor Siegbahn's laboratory.[2] The separation of these components of the Kβ line from each other amounts to a small fraction of 1%.

According to Rubinowicz' "Principle of Selection" transfers of electrons from the first M orbit to the K orbit ought not to occur.[3]

The faint component that has been observed by Siegbahn,[4] de Broglie[5] and Hjalmar on the long wave-length side of the Kβ line in the spectra of a few of the chemical elements was predicted by Sommerfeld[6] by reasoning from his theory of elliptic orbits.

These components of the Kβ line may account for some of the irregularities in the β peaks on our curves (figs. 2 and 3).

Some years ago Kossel suggested that the difference between the frequencies (or wave-numbers) of the Kβ and Kα lines should equal that of a line in the L series. It is well known that this relation, although approximately true, is not quite exact. An explanation for this discrepancy may be found in the supposition that the electrons producing the Kβ line may not come from the same M orbits as those producing the Lα and Lβ lines.[7] Dr. R. A. Patterson and one of us recently presented to the American Physical Society an account of some experiments which show that the wave-numbers of the $L\alpha_1$, $L\alpha_2$ and $L\beta_1$ lines of uranium and thorium equal the differences between certain of the L and M critical absorption wave-numbers. These relations may be expressed by the following equations:

$$La_1 - Ma_1 = L\alpha_1,\ La_1 - Ma_2 = L\alpha_2 \text{ and } La_2 - Ma_2 = L\beta_1.$$

In terms of the transfer of electrons between orbits these equations mean that the $L\alpha_1$, $L\alpha_2$ and $L\beta_1$ lines are due to electrons falling from the M_1 and M_2 orbits into L orbits, whereas we inferred above that the $K\beta$ line was mainly due to electrons falling from the M_3 orbit into the K orbit. A rough quantitative test for this explanation may be obtained as follows: From the above three equations and the three equations representing the transfers of electrons producing the $K\alpha$ and $K\beta$ lines, namely

$$Ka - La_1 = K\alpha_1,\ Ka - La_2 = K\alpha_2 \text{ and } Ka - Ma_3 = K\beta,$$

we get the equations

$$K\alpha_1 + L\alpha_1 - K\beta = Ma_3 - Ma_1 \text{ and } K\alpha_2 + L\beta_1 - K\beta = Ma_3 - Ma_2.$$

The left-hand members of these equations have the values 0.043 and 0.039, respectively, for tungsten. The right-hand members, which are the differences between quantities that we have estimated by extrapolation, have the values 0.030 and 0.025, respectively. These small differences are of the same order of magnitude, which indicates that the explanation is approximately correct. We must remember, however, that the M series is probably more complicated than we have supposed. There are reasons for believing, for instance, that another critical absorption exists, of slightly higher frequency than Ma_3.

The ionization spectrometer furnishes us a method of approximately estimating the relative intensity of the lines in X-ray spectra. At present, however, we are not able to correct for the change with wave-length in the absorption of the rays by the target, the glass walls of the X-ray tube and ionization chamber, the reflecting crystal, etc. Nor do we know exactly how the coefficient of reflection of the crystal varies with the wave-length. It is not safe, therefore, to compare spectral lines that differ very much from each other in wave-length.

By estimating the heights of the peaks corresponding to the lines of the K series in the spectra of the first, second, third and fourth orders we have arrived at the following numbers representing approximately their relative intensities:

Line........:........	α_3	α_2	α_1	β	γ
Intensity............	4	50	100	35	15

It is interesting to note that, if we apply Sommerfeld's theory of the relative intensity of lines (Atombau und Spektrallinien, chapter 6) to the α_1 and α_2 lines, we get either 3 : 2, or 2 : 1, as the ratio of their intensities. Our experiments favor the latter ratio.

In an address delivered at the St. Louis meeting of the American Association for the Advancement of Science, last December (see *Science*, May 21, 1920, p. 505) one of us presented a set of calculations of the K critical absorption frequencies for a number of chemical elements. The calculations were based on Bohr's theory, with the additional assumption that the electrons were distributed among the orbits in much the same way as they are distributed in the layers and shells of the Lewis-Langmuir static atom. The ratio, ν/ν_∞, of the K critical absorption frequency for tungsten to the Rydberg fundamental frequency is 5118 according to the theory, which is exactly our experimental value (see table 1). The agreement to four figures between the two values must be regarded as a matter of chance, however, as the errors in both the experimental and calculated numbers amount to a fraction of 1%.

[1] See Sommerfeld's paper on the fine structure of the β-line (*Münchener Akademie,* June 1, 1918 (367)).

[2] Professor Siegbahn kindly communicated these results by letter.

[3] *Physik. Zs.*, **19**, 1918 (441, 465).

[4] *Phil. Mag.*, June, 1919 (601).

[5] *Comptes Rendus*, May 25, 1920 (1245).

[6] *Sitz. Ber. Bayr. Akad. Wiss.*, June, 1918.

[7] Since this explanation was presented to the Physical Society in February, papers by W. Kossel and A. Sommerfeld (*Zeitschrift für Physik*, Jan., 1920) have reached us. In these papers the same explanation for the discrepancy is discussed as that here put forward.

STUDIES OF MAGNITUDE IN STAR CLUSTERS, XII. SUMMARY OF A PHOTOMETRIC INVESTIGATION OF THE GLOBULAR SYSTEM MESSIER 3[1]

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Communicated by G. E. Hale, June 24, 1920

1. The study at Mount Wilson of the distances and structure of stellar clusters is of necessity based upon observations extensive in number and considerably varied in character. When general and comprehensive results have been obtained for a few typical systems, it becomes possible to interpret more accurately the less detailed observations on other clusters, and to place greater confidence in the hypothesis of the galactic system that has developed from the investigations of cluster and variable stars.

For two open clusters, Messier 67 and 11, and for two globular clusters, Messier 13 and 3, fairly complete surveys of the photographic and photovisual magnitudes of the individual stars have now been carried through and published in some detail; for about forty other systems the photometric and spectral investigations have been of a special character, and, in publishing the results, the observational data have for the most part only been summarized.

2. The catalogue of magnitudes and colors of 848 stars in Messier 3, which is appearing as *Mount Wilson Contribution* No. 176,[2] was begun in 1915. Fourteen photographs, made at the primary focus of the 60-inch telescope, have been used for the ten thousand measures of magnitude involved in the investigation. Special care was taken to look into the